Dynamics of a large ocean vessel

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1 Introduction

Knowledge about wave induced loads and motions of ships and offshore structures is important both in design and operational studies. Due to the economic meaning of oil exploration or transport systems and the risk of accidents endangering human lives and the sensitive environment, the prediction and characterization the dynamic behavior of these systems is essential. Determining or even increasing operation ranges is generally a non-trivial task.

A ship moored at a wharf or anchored to the ocean floor may be acted on by a variety of dynamic environmental forces such as

- Wind
- Current
- Passing vessel effect
- Wave action
- Tidal variations and vessel draft changes
- Ice

These forces cause the ship to undergo motions which may be sufficiently large and thus may not only negatively affect port operations (loading and unloading) but even result in damage to wharf and ship, or loss of life. The situation with large ships such as oil vessels or giant 'mother ship platforms' is more difficult, since these ships carry a rather large momentum even at relatively small velocities while the stopping distance of tankers are typically on the order of kilometers. In the case of 'mother ship platforms' used for oil storage out at sea the flexible hose connecting the ship to the well-head may break causing an environmental disaster. So, in addition to staying secured in its berth, a moored vessel must remain as motionless as possible within a limited range of acceptable motions.

In a three-dimensional Cartesian coordinate system rigid body can undergo six motions — three translations and three rotations. In case of a floating structure the translatory motions are referred to as surge, sway and heave, with heave being the vertical motion (see Fig. 1). The angular motions are referred to as roll, pitch and yaw, with yaw being rotation about a vertical axis.

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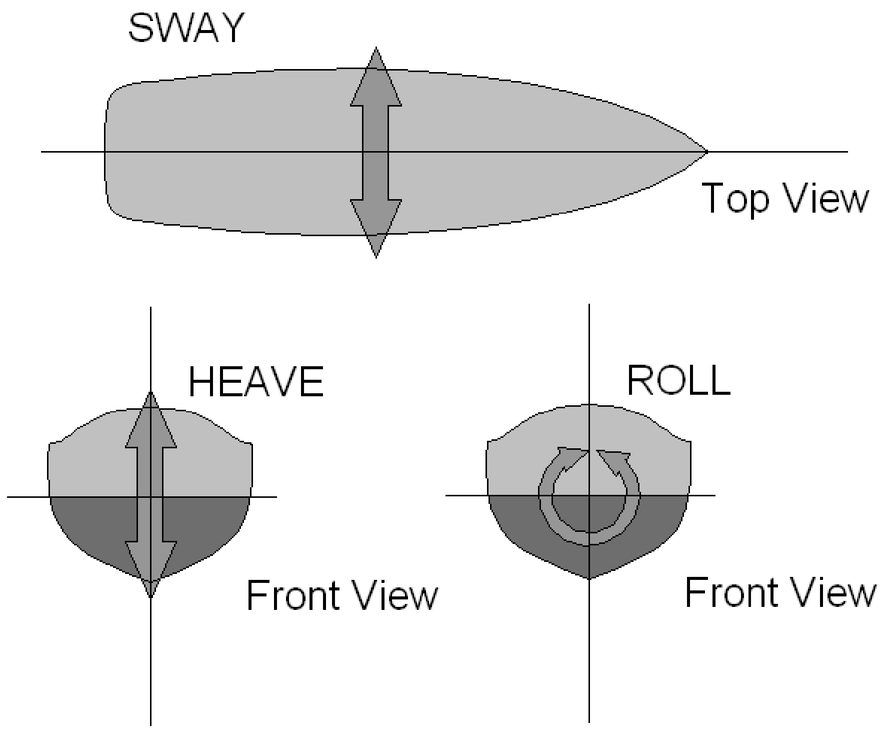

Figure 1: Motions of a ship

As mentioned above, studying the dynamics of marine vessels is necessary for attempting to engineer ways of keeping a ship stable and prevent potential disasters. There are different approaches to modelling the dynamics of a moored ship under the action of external forces. Generally a non-linear system of coupled ODE's of motions arises and can be studied using linearization technique [3], [1], bifurcation theory [1] and numerical calculations [1], [2]. Underlying assumptions of a model as well as simplifications done during its further investigation are crucial.

The aim of the present study is to obtain a rather simple, yet reliable, model of a ship in an open ocean under the action of a periodic incoming wave. The general arrangement and basic assumptions of the model are given in Section 2. These assumptions lead to a non-linear system of coupled differential equations which describe the motion of the ship. Approximate solutions are obtained in Section 3 by first nondimensionalizing the governing equations and then applying perturbation theory. A discussion of the key results, which include fundamental frequencies and typical amplitudes of motion, are presented in Section 4, which is followed by a brief summary.

2 Model

When a large ship is moored, it is being pressed against a dock typically for the loading and unloading of cargo or passengers. There are several complicated dynamics involved in the mooring which are either natural or introduced by people. The wind, water currents, and water waves create forces on the heavy ships which cause both translational motion and rotational motion. To help counteract these effects, fenders and mooring ropes are put in place to prevent damage to the ship, to the dock, and to maintain overall safety. To begin an elementary modelling investigation, it is appropriate to first look solely at the effects of waves against a ship.

2.1 Assumptions

The following assumptions have been made to the model for a simplified basis and to capture the essence of the physics.

- Consider the ship to be a hollow rectangular prism with height H in the y direction, length L in the z direction, and berth B in the x direction.
- The ship's mass is uniformly distributed along the six faces and any material stored inside the ship is assumed to be a motionless solid (relative to the ship) with uniform density.
- For water height reference, the top of the water line is considered to be at $y = 0$. In reference to this spot, the centre of mass of the ship is considered to be a vertical distance y away from the water line.
- Periodic waves with (angular) frequency ω and maximum height h from the top of the water line strike the ship from the left side. Therefore, under this assumption, the height of the water wave can be modelled as
$$y_w = h \sin \omega t. \tag{1}$$
A sine wave is chosen so the ship is initially in dynamical equilibrium and free of any external forces.
- The water waves extend infinitely in the z direction with a uniform height and are fully absorbed by the ship (i.e. there is no wave transmission or reflection). Therefore there is only still water on the right side. To maintain continuity, a linear depth is assumed for the distance that extends from the wave height on the left to the still water on the right (Figure 2).
- Since the waves are uniform in the z direction this means that only two dimensional motion in the x (sway) direction and in the y (heave) direction is considered. This also means that the only rotational motion will be about the z axis (roll).
- The pressure that the water exerts on the ship is hydrostatic. Based on this assumption, anything at a depth s below the surface of the water will experience a pressure
$$p = \rho g s$$
where ρ is the (uniform) density of the water and g is the acceleration due to gravity.

2.2 Ship Geometry

Figure 2 shows a schematic diagram of the ship as the waves act on it. The bottom of the ship is marked B to represent its berth and the solid black circle represents the centre of mass of the ship a distance y from the surface of the water. The heave represents the vertical distance this centre of mass travels in time. The physical location of the centre of mass with respect to the ship does not change with time as no mass is being added or removed and it is a distance d from the bottom

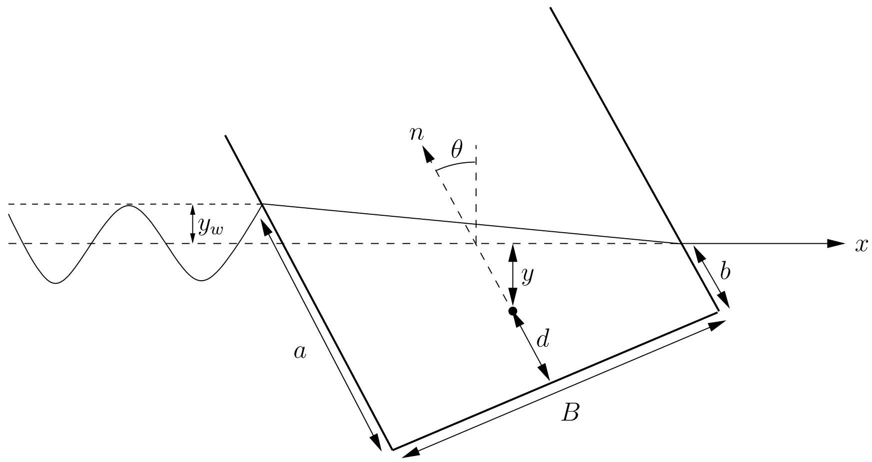

Figure 2: Schematic diagram of the model, showing the ship (thick-solid), the water level (thin-solid), and key distances used to formulate the governing equations. The black dot represents the centre of mass of the ship and the height of the wave, y_w, is given by (1). For clarity, the diagram depicts the horizontal distance travelled, x, as being measured relative to the right side of the ship. However, in the governing equations, this distance is actually measured relative to the *left* of the ship.

of the ship. The variable x is measured from the left edge of the ship so that the sway motion represents the distance the left-edge has moved from its original position in time. A line normal to the bottom of the ship, n, extends outwards and makes an angle θ with the y-axis. Because of the rectangular geometry, the sides of the ship are parallel to this normal and therefore make the same angle with the y-axis. The amount of the left side of the ship that is submerged under water is denoted a and as can be seen from the figure,

$$a\cos\theta = h\sin\omega t + y + d\cos\theta + \frac{B}{2}\sin\theta$$

and similarly the right side of the ship that is submerged under water is denoted b and is defined as

$$b\cos\theta = y + d\cos\theta - \frac{B}{2}\sin\theta.$$

It was an assumption that the depth of the water from the left side where the wave crest hits to the right side where the water is still (Figure 2) is modelled linearly. The height of the water on the left surface is the height of the water wave at that time, $h\sin\omega t$, and the height of the water at the right surface is zero. Using a locally defined coordinate system that lies along this linear water profile, it can be shown that the left surface occurs at $u = 0$ and the right surface occurs when $u = B\cos\theta + (a-b)\sin\theta$. Therefore, the height of this linear water profile, h_l, relative to the neutral water line ($y = 0$) is

$$h_l(u) = h\sin\omega t\left[\frac{B\cos\theta + (a-b)\sin\theta - u}{B\cos\theta + (a-b)\sin\theta}\right]. \tag{2}$$

2.3 Center of Mass

Since the ship is a hollow rectangular box with mass m then the centre of mass position of it is $\left(\frac{B}{2}, \frac{H}{2}, \frac{L}{2}\right)$. The fluid being carried inside the ship is chosen to be oil having a uniform density $\bar{\rho}$ and it is evenly distributed throughout the length and berth of the ship. The height of the fluid is η making the centre of mass position of the fluid $\left(\frac{B}{2}, \frac{\eta}{2}, \frac{L}{2}\right)$. The centre of mass vector is then defined to be

$$\mathbf{x}_{\text{cm}} = \frac{1}{2}\frac{m\,(B, H, L) + \bar{\rho}LB\eta\,(B, \eta, L)}{m + \bar{\rho}LB\eta}.$$

In terms of centre of mass, the bottom corner of the ship is assumed to be the origin, the y component of the centre of mass represents d, the distance from the bottom of the ship to the centre of mass. Therefore,

$$d = \frac{1}{2}\frac{mH + \bar{\rho}LB\eta^2}{m + \bar{\rho}LB\eta}. \tag{3}$$

2.4 Forces

The force on each side will just be the hydrostatic pressure pushing on that particular side. The force is then

$$F = \iint p \, \mathrm{d}A = \iint \rho g s \, \mathrm{d}A$$

where $\mathrm{d}A$ is an area element of the surface that the water is acting on. The force acts in a direction normal to this surface. For the left side of the ship, the force is acting on the surface of length L in the z direction and width a in a direction that can be denoted q. The force on this side then is

$$F_L = \rho g L \int_0^a s(q)\,\mathrm{d}q \tag{4}$$

where the problem simply is to find the depth s as a function of q. Note from Figure 2 that if the height of the wave extended in a straight line to the right side, the depth at any q would simply be $s = q\cos\theta$. However, the linear wave profile causes a gap of $h\sin\omega t - h_l(u)$ and therefore, the depth is

$$s(q) = q\cos\theta - h\sin\omega t + h_l(u)$$

where $u = q\sin\theta$. Substituting (2) for h_l, the above equation for s becomes

$$s(q) = q\left[\cos\theta - \frac{h\sin\omega t\tan\theta}{B + (a-b)\tan\theta}\right]. \tag{5}$$

Substituting (5) into (4), the force on the left side of the ship is

$$F_L = \frac{1}{2}\rho g L a^2 \frac{B\cos\theta}{B + h\sin\omega t\sin\theta}. \tag{6a}$$

The forces on the remaining sides can be determined in a similar way by finding a different function for the depth. The forces on the right, F_R, and the bottom, F_B, are

$$F_R = \frac{1}{2}\rho g L b^2\cos\theta \tag{6b}$$

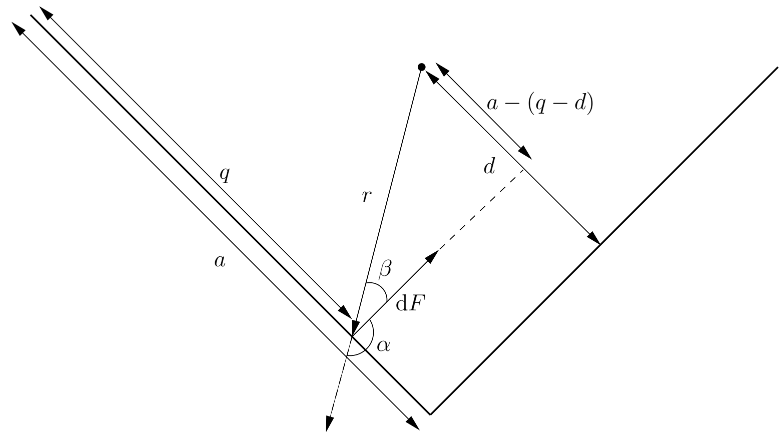

Figure 3: Illustrating the geometry required to compute the torque on the left wall of the ship. The black dot represents centre of mass of the ship.

and

$$F_B = \rho g L B (y + d\cos\theta) + \frac{1}{2}\rho g L h \sin\omega t \frac{(B\cos\theta - b\sin\theta)^2}{h\sin\omega t\sin\theta + B}, \tag{6c}$$

respectively, where again, the direction of each force is normal to the respective surface.

2.5 Torques

Since the three forces act along each point of their respective surface and not just the centre of mass of the ship, the ship will experience a nonzero net torque and undergo rotational motion about its centre of mass. The torque is formally defined as

$$\tau = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \int r\rho g s \sin\alpha \,\mathrm{d}q$$

where α is the angle between the differential force $\mathrm{d}\mathbf{F}$ and the position vector $\mathbf{r}$ when the vector tails are touching. The magnitude of $\mathbf{r}$ is denoted by r, which is the distance of each force component to the axis of rotation which is about the centre of mass. The torques are defined so that counter-clockwise is positive. Figure 3 shows an example force on the left side of the ship. Because of the ambiguity of sine,

$$\sin\alpha = \sin\beta = \frac{d-(a-q)}{r}$$

and therefore

$$\tau_L = \rho g L \int_0^a s(q)\,(d-a+q)\,\mathrm{d}q. \tag{7}$$

However, (5) may be used again since this is the left side and substituting this into (7),

$$\tau_L = \frac{1}{2}\rho g L a^2 \left(\frac{B\cos\theta}{B + h\sin\omega t\sin\theta}\right)\left(d - \frac{a}{3}\right). \tag{8a}$$

If $d > a/3$ then the angle is shallow and more water is below the centre of mass creating a counter-clockwise torque and if $d < a/3$ then the angle is steep and more water is above the centre of mass creating a clock-wise torque. In this case, a negative sign will be produced, correctly adhering to the definition of positive torque stated. In a similar fashion to the forces, the torque on the right surface, τ_R, and the torque on the bottom surface, τ_B, can be calculated and are given by

$$\tau_R = \frac{1}{2}\rho g L b^2 \cos\theta \left(\frac{b}{3} - d\right) \tag{8b}$$

and

$$\tau_B = -\frac{1}{12}\rho g L \left[B^3 \sin\theta + h\sin\omega t\,(B\cos\theta + 2b\sin\theta)\,\frac{(B - b\tan\theta)^2}{(B + h\sin\omega t\sin\theta)}\right] \tag{8c}$$

respectively.

2.6 Equations of Motion

From Figure 2 along with equations (6) and (8), the components of forces and torques putting sway, heave, and roll motion on the ship are

$$m\ddot{x} = F_L\cos\theta - F_R\sin\theta - F_B\sin\theta \tag{9a}$$

$$m\ddot{y} = F_R\sin\theta - F_L\sin\theta - F_B\cos\theta + mg \tag{9b}$$

$$I\ddot{\theta} = \tau_R + \tau_L + \tau_B \tag{9c}$$

where the appropriate signs in the torques have been taken care of by their definitions. To maintain dynamical equilibrium at $t = 0$, the following initial conditions are imposed,

$$x(0) = \dot{x}(0) = 0, \quad y(0) = \frac{m}{\rho L B} - d,\ \dot{y}(0) = 0, \quad \theta(0) = \dot{\theta}(0) = 0.$$

Parameter values for this model are given in Table 1. Many of the parameters were specified in the problem or chosen in the model without loss of generality.

2.7 Scaling and Nondimensionalization

The governing equations presented above can be simplified by making use of the fact that the height of the incoming wave is very small compared to the physical dimensions of the ship. Furthermore, the moment of inertia of the ship is sufficiently large that under non-resonance conditions the ship's roll is on the order of a couple of degrees. Thus, the small angle approximation can be used to simplify the trigonometric terms that appear in the equations. To be precise, the angle θ is scaled according to $\theta \to \varepsilon\theta$, where ε is a small nondimensional number given by

$$\varepsilon = \frac{h}{B} \ll 1.$$

The x variable is nondimensionalized by the scaling $x \to Bx$, and the small angle approximation allows the y variable to be scaled via $y \to Hy - d$. Furthermore, time is scaled by $t \to \tau t$, where

$$\tau = \left(\frac{2m}{\rho g L B}\right)^{1/2}$$

Table 1: Description of model parameters and their values.

Parameter	Interpretation	Value	Reference
L	length of ship in z direction	350 m	problem specified (PS)
B	berth or width of ship in x direction	50 m	PS
H	height of ship in y direction	100 m	PS
η	height of oil in ship	30 m	model chosen (MC)
d	distance of centre of mass from bottom of ship	28.1 m	(3)
ρ	density of water	1000 kg/m^3	equation [5]
$\bar{\rho}$	density of oil	800 kg/m^3	[4]
h	maximum wave amplitude height	1 m	PS
ω	angular frequency of incoming wave	0.314 1/s	PS
m	mass of ship	2.5×10^8 kg	PS
I	moment of inertia of ship about an axis parallel to the z-axis through its centre of mass	1.0×10^{10} kg·m^2	MC
g	gravitational constant of Earth	9.81 m/s^2	[5]

is a time constant. With this scaling, the governing equations can be written as

$$\ddot{x} = \varepsilon[X_1(x,y,\theta,t) - 2\alpha y\theta] - \varepsilon^2[X_2(x,y,\theta,t) + X_3(x,y,\theta,t)], \tag{10}$$

$$\alpha\ddot{y} = P - 2\alpha y - \varepsilon[Y_1(x,y,\theta,t) - Y_2(x,y,\theta,t)], \tag{11}$$

$$k\varepsilon^2\ddot{\theta} = \Theta_1(x,y,\theta,t) - \Theta_2(x,y,\theta,t) + \varepsilon\Theta_3(x,y,\theta,t), \tag{12}$$

where $\alpha = H/B \sim O(1)$ is the aspect ratio of the ship and

$$P = \frac{2m}{\rho L B^2}, \quad k = \frac{I}{mB^2\varepsilon}$$

are order one nondimensional numbers. The overdot now represents differentiation with respect to the nondimensional time t. The functions X_i, Y_i, and Θ_i that appear in (10) - (12) are given by

$$\begin{aligned}
X_1 &= (\sin\Omega t + \theta)(\varepsilon\sin\Omega t + 2\alpha y),\\
X_2 &= [\alpha y + \varepsilon(\sin\Omega t + \theta/2)]^2[1 - \varepsilon^2\theta\sin\Omega t]\theta\sin\Omega t,\\
X_3 &= [1 - \varepsilon^2\theta\sin\Omega t][1 - \varepsilon(\alpha y - \varepsilon\theta/2)\theta]^2\theta\sin\Omega t,\\
Y_1 &= \left\{[\alpha y + \varepsilon(\sin\Omega t + \theta/2)]^2[1 - \varepsilon^2\theta\sin\Omega t] - 1\right\}\theta,\\
Y_2 &= [1 - \varepsilon^2\theta\sin\Omega t][1 - \varepsilon(\alpha y - \varepsilon\theta/2)\theta]^2\sin\Omega t,\\
\Theta_1 &= [\alpha y - \varepsilon\theta/2]^2[(\alpha y - \varepsilon\theta/2)/3 - d/B],\\
\Theta_2 &= [\alpha y + \varepsilon(\sin\Omega t + \theta/2)]^2\left\{[\alpha y + \varepsilon(\sin\Omega t + \theta/2)]/3 - d/B\right\},\\
\Theta_3 &= \left\{\theta + [1 - \varepsilon^2\theta\sin\Omega t][1 - \varepsilon(\alpha y - \varepsilon\theta/2)\theta]^2[1 + 2\varepsilon(\alpha y - \varepsilon\theta/2)\theta]\sin\Omega t\right\}/6,
\end{aligned}$$

where $\Omega = \tau\omega$ is the nondimensional angular frequency of the incoming wave. Initial conditions for the rescaled system are as follows,

$$x(0) = \dot{x}(0) = 0, \quad y(0) = \frac{P}{2\alpha}, \; \dot{y}(0) = 0, \quad \theta(0) = \dot{\theta}(0) = 0. \tag{13}$$

Examining the rescaled equations (10) - (12) can yield fruitful insight into dynamics that govern the motion of the ship. To leading order the heaving motion (the motion in the y direction) is the simple harmonic bobbing motion that occurs when there is an inbalance between the gravitational force and the buoyancy force. The effects of rolling can be considered as a higher order correction to the bobbing motion. The equation for the roll suggests that θ is changing on multiple time scales, namely the time scales τ and τ/ε. The primary rolling motion of the ship is caused by a torque inbalance across the width of the ship, which occurs on the fast τ/ε time scale. Small corrections to this motion occur on the τ time scale.

3 Multiple Scales and Perturbation Theory

To resolve both time scales in the rolling motion, the variable θ is written as $\theta = \theta(t, T)$, where $T = t/\varepsilon$ is a fast (nondimensional) time. The total time derivative of θ is now given by

$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + \frac{1}{\varepsilon}\frac{\partial\theta}{\partial T},$$

which separates the two time scales. The heave and the sway motions of the ship occur on the regular time scale and thus multiple time scales are not required for x and y. However, these functions are expanded as a power series in ε,

$$\begin{aligned} x(t,\varepsilon) &= \varepsilon x_1(t) + \varepsilon^2 x_2(t) + O(\varepsilon^3), \\ y(t,\varepsilon) &= y_0(t) + \varepsilon y_2(t) + O(\varepsilon^2). \end{aligned} \tag{14}$$

Loosely speaking, the order one term in x is not required because the leading order terms on the right hand side of (10) and the initial conditions for x are $O(\varepsilon)$. Inserting solutions of this form into (12) and collecting powers of ε yield

$$\begin{aligned} O(1): &\qquad \frac{\partial^2\theta}{\partial T^2} = 0, \\ O(\varepsilon): &\qquad \frac{\partial^2\theta}{\partial T\partial t} = -\Gamma(\theta + \sin\Omega t), \end{aligned}$$

where $\Gamma = \Gamma(B, P, d)$ is a positive constant. The first equation implies that θ is a linear function of the fast time variable T with slowly varying coefficients, $\theta = c_1(t)T + c_2(t)$. Inserting this solution into the $O(\varepsilon)$ equation shows that $c_1(t) \equiv 0$. Therefore, the roll of the ship is approximately given by

$$\theta = -\sin\Omega t. \tag{15}$$

Equations for the sway and the heave motion can be obtained using the same method as above. The heave equations are given by

$$\begin{aligned} O(1): &\quad \alpha\ddot{y}_0 = -2\alpha y_0 + P, \\ O(\varepsilon): &\quad \alpha\ddot{y}_1 = -2\alpha y_1 - (\alpha^2 y_0^2 - 1)\theta - \sin\Omega t, \end{aligned}$$

with initial conditions $y_0(0) = P/2\alpha$, $y_1(0) = 0$, and $\dot{y}_0(0) = \dot{y}_1(0) = 0$. The condition for $y_0(0)$ results from the fact that $y(0) = P/2\alpha \sim O(1)$. Since θ is known from (15), solutions to this system are straightforward to obtain and are given by

$$\begin{aligned} y_0(t) &= \frac{P}{2\alpha}, \\ y_1(t) &= \frac{1}{8\alpha}\left(\frac{P^2-8}{\Omega^2-2}\right)\left[\sqrt{2}\sin(\sqrt{2}t) - 2\sin(\Omega t)\right]. \end{aligned} \tag{16}$$

The sway equations to second order read

$$\begin{aligned} O(\varepsilon): &\quad \ddot{x}_1 = 2\alpha y_0 \sin\Omega t, \\ O(\varepsilon^2): &\quad \ddot{x}_2 = 2\alpha y_1 \sin\Omega t + (1+\alpha^2 y_0^2)\sin^2\Omega t, \end{aligned}$$

where (15) has been used to simplify the $O(\varepsilon^2)$ equation. The initial conditions are $x_i(0) = \dot{x}_i(0) = 0$ for $i = 1, 2$. Upon substitution of (16), solutions to the above equations can be obtained by direct integration, resulting in

$$\begin{aligned} x_1(t) &= \frac{P}{\Omega^2}\left(\Omega t - \sin\Omega t\right), \\ x_2(t) &= \frac{\gamma_1}{2}\left\{\frac{\cos[(\sqrt{2}+\Omega)t]-1}{(\sqrt{2}+\Omega)^2} - \frac{\cos[(\sqrt{2}-\Omega)t]-1}{(\sqrt{2}-\Omega)^2}\right\} + \frac{\gamma_2}{4}\left\{t^2 + \frac{\cos(2\Omega t)-1}{2\Omega^2}\right\}, \end{aligned} \tag{17}$$

where γ_1 and γ_2 are constants that are given by

$$\gamma_1 = \frac{\Omega}{\sqrt{8}}\left(\frac{P^2-8}{\Omega^2-2}\right), \quad \gamma_2 = \frac{P^2}{4} + 1 - \frac{\sqrt{2}}{\Omega}\gamma_1.$$

These solutions, along with numerically approximated solutions to the full system of nonlinear equations, are shown in Figure 4.

4 Discussion

From Figure 4, it is clear that the analytical solution that is obtained via perturbation methods captures the general motion of the ship. In fact, the sway and heave motions appear to be captured nearly perfectly. As expected from the analysis, the solution for θ fails to capture the small scale variations that occur on the time scale of τ, which is approximately three seconds for the parameters in Table 1. To capture this motion, a higher order analysis would have to be performed. However, the solution obtained does contain several interesting features. In particular, the ship rolls with the same frequency as the incoming wave, and the amplitude of these oscillations is $\varepsilon = h/B$.

For a short amount of time, on the order of a couple of waves, the $O(\varepsilon)$ solution to the sway equation (10) is a reasonable approximation (see Figure 5). At this order, the ship has a linear drift in time that has a small sinusoidal variation superimposed on it. It can be shown that the ship drifts a (dimensional) distance of

$$x_{\text{drift}} = \frac{2\pi g}{\omega^2}\frac{h}{B}$$

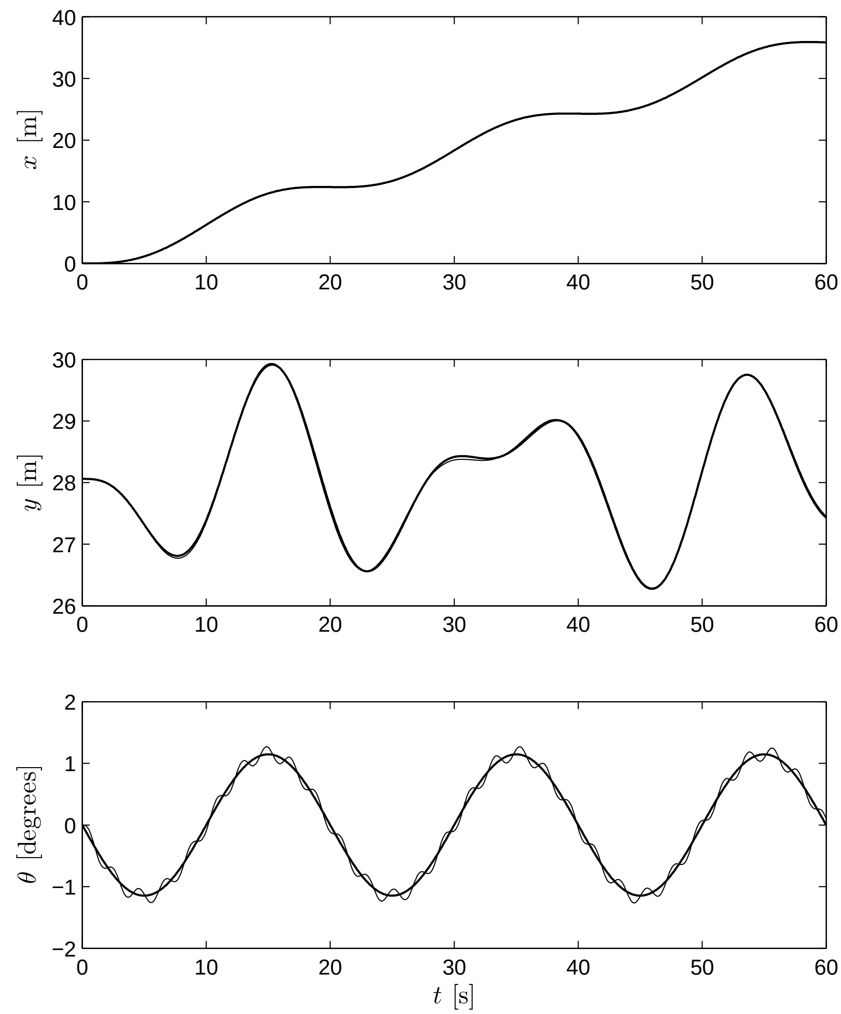

Figure 4: Solutions (in dimensional form) to the governing equations. The thick curves are solutions obtained via perturbation methods and the thin curves are solutions obtained via numerical integration. From top to bottom, the figures show the sway, the heave, and the roll.

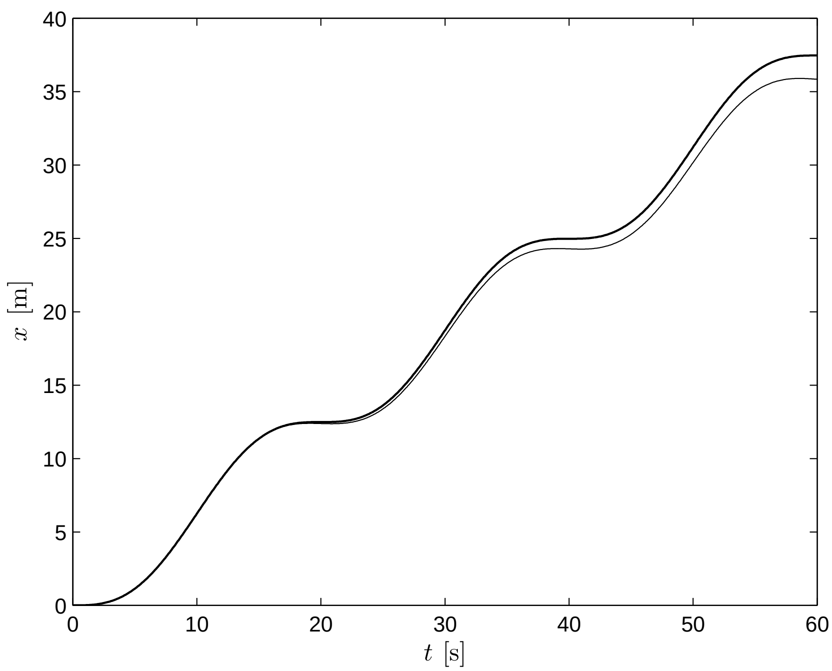

Figure 5: The approximate sway solution (thick) using only the $O(\varepsilon)$ term. Also shown is the numerical solution (thin).

for each wave that passes. This is a somewhat surprising result because it is independent of all but one of the ship's properties, namely the berth. Thus, to minimize the sway of a ship, the berth should be as large as possible. The approximation at $O(\varepsilon)$ is unable to capture the values at larger time because of the quadratic term in the x_2 solution [see (17)].

The approximate solution to the heave equation (11), given by (16), contains two notable features. Firstly, resonance phenomena can occur if the incoming wave has (dimensional) frequency

$$\omega_{\text{res}} = \left(\frac{\rho g L B}{m} \right)^{1/2}$$

and secondly, at second order it is possible to eliminate heave if

$$\frac{m}{\rho L B^2} = \sqrt{2}.$$

However, this does not necessarily imply that heaving motion will be completely eliminated. For example, having this equality satisfied might be outside of our model. That is, terms which are currently negligible could become large and make an appearance in the lower order equations. Furthermore, the equation for y_1 (16) can be used to estimate the amplitude of the heaving motion.

5 Summary

The motion of a large ship subject to a small incoming wave has been analyzed. Using laws from classical mechanics, a set of cumbersome nonlinear equations could be obtained which modelled the sway, the heave, and the roll of the ship. The equations were nondimensionalized rescaled into a form that was suitable for the application of perturbation theory, where the small number was the ratio of wave height to the berth of the ship. The solutions that were obtained with this method are in excellent agreement with those obtained via numerical integration. Furthermore, analyzing the solutions revealed several key relationships between the motion of the ship and the physical parameters of the problem.

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